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RBFs with Spatially Variable Shape Parameters and Optimized Knot Locations

F. L. Zhou[1]*, J. M. Zhang [2]

[1 2] *State Key Laboratory of Advanced Design and Manufacturing for Vehicle Body, Hunan University, Changsha, 410082 China*
[1] Email: edwal0zhou@gmail.com
[2] Email: zhangjianm@gmail.com

Abstract

Radial Basis Function (RBF) with spatially variable shape parameters has been studied in attempt to improve the stability of the interpolation. The RBF interpolation, as we know, is quite efficient and has a wide range of application. However, the RBF interpolation suffers from a contradiction between accuracy and stability, which can be expressed in a form similar to the Uncertainty Principle in quantum mechanics [1].

For global RBFs, the shape of the function is usually controlled by a parameter c. A proper value of c is crucial importance for the stability and accuracy of RBF interpolation, as the condition number of the interpolant matrix may varies exponentially as the shape parameter changes. On the other hand, the exponentially better accuracy can be obtained by choosing a proper c [4]. In the literatures, most RBF researchers emphasize more on accuracy than on stability. For engineering application, however, the stability of the RBF interpolation should be more important, as the relative error can not be evaluated without an exact solution, which is often unavailable for practical problems. In this paper we study the sensitiveness of the condition number of the interpolant matrix to the parameter c. One way suggested in many literatures to circumvent the ill-condition problem is to use spatially variable shape parameter [3]. Although using spatially variable shape parameter may lead to singular interpolation matrix [2], the non-singularity of an interpolation matrix can be guaranteed by some special parameter variation strategies.

The Runge phenomenon, which is best known in high-order polynomial interpolation, also may appear in the RBF interpolation [2]. Large interpolation error often occurs at locations near the boundary. For engineering application, however, accurate interpolation near the boundary is important. Further more, as we know, oscillatory behavior is usually seen in high order Lagrange interpolations when the original function has a jump discontinuity. This is called the Gibbs Phenomenon. The RBF interpolation, especially the global ones, also suffers from the Gibbs Phenomenon. In fact, the function with jump discontinuity can be considered as a special case of the ones that have a large value of the derivative. By varying the shape parameters of the RBF interpolation, both of the Runge phenomenon and the Gibbs Phenomenon can be suppressed.

To circumvent the ill-condition problem, many methods have been proposed [3]. These methods can be sorted into seven classes:

1. simple preconditioners

2. variable shape parameter

3. transform the global basis function into a truncated basis function

4. the multi-zone method

5. optimization of knot location

6. multilevel approximation schemes

7. block substructuring and partitioning

The concept of variable shape parameters in the RBF interpolation has been proposed by many researchers, such as E. J. Kansa [3], M. D. Buhmann, et al. The main idea is to use different values of shape parameters for the RBFs that with different knot locations. The shape parameters are determined according to the density of RBF knots in a local area. In the optimization of knot location method, the interpolation points are adjusted to follow the peak of the shock wave. The optimization may yield coefficient matrix with lower condition number and better accuracy with even a small number of interpolation points.

The methods of variable shape parameter and the optimization of knot location were combined in this paper. The error bounds and the stability of the combined approach were studied in detail. Mathematical proofs for the convergence and stability were presented. In order to find a stable and efficient interpolation scheme, several kinds of 3D RBFs and parameter variation strategies were compared in the numerical examples. Results show that the combined approach is stable and possesses high accuracy. Moreover, in contrast with conventional RBF interpolation, additional interpolation points can be added to improve the accuracy without much loss of stability in our method.

The Boundary Face Method [5] has been combined with the Dual Reciprocal Method to solve 3-D inhomogeneous potential problems. We call the combined method as the Dual Reciprocal Boundary Face Method (DRBFM). In the DRBFM, the combined interpolation approach has been implemented. Numerical examples which demonstrate the stability and accuracy are also presented.

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